Utilizing generalised trapezoidal fuzzy numbers, a novel ranking method

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ABSTRACT

Fuzzy numbers play a crucial position in decision making, optimization, forecasting, and other fields of study. Prior to taking action, hazy numbers must be assessed by an executive. The ranking technique proposed by Chen and Chen (Expert Systems with Applications 36 (2009) 6833-6842) is proven to be erroneous in this study utilizing multiple counter cases. In this research, we explore several cutting-edge techniques for ordering fuzzy generalized trapezoidal numbers. The proposed method is helpful since it helps to sort generalized and normal trapezoidal fuzzy numbers in the correct order. All of the necessary characteristics of fuzzy quantities are present in the proposed ranking function, as stated by Wang and Kerre (Fuzzy Sets and Systems 118 (2001) 375-385).

Keywords:

Generalized trapezoidal fuzzy numbers, and the ranking function.

INTRODUCTION

UZZY set theory [1] has the potential to provide useful solutions to real-world issues. Though or may be used to sort real numbers, it has no such effect on fuzzy ones. It is difficult to establish if one fuzzy number is more or smaller than another as fuzzy numbers are represented by a range of potential outcomes. Using a ranking function to sort the fuzzy integers is an effective method. The set of fuzzy numbers (F (R) R) is defined in terms of real numbers, and each fuzzy number is then ordered along the real line. Fuzzy set theory has become increasingly concerned with the precise ordering of fuzzy numbers, which is a crucial step for making judgments in a fuzzy environment.

For Jain, ranking was an original concept. In [0,1], Yager [3] suggested four indices that may be used to sort fuzzy values. Fuzzy number sorting is addressed in Kaufmann and Gupta [4]. [5] Campos and Gonzalez [5] offered a subjective technique of assessing fuzzy integers. Liou and Wang [6] devised an integral value index. For fuzzy integers, Cheng [7] presented a ranking system based on distance. The similarities between Kwang and Lee are many.

Fuzzy number probability distributions were used to build a ranking method by [8]. Modarres and Nezhad [9] introduced a preference function-based ranking method, in which the fuzzy numbers are evaluated incrementally, with the most preferred value being determined at each stage. Chu and Tsao [10] claim that Fuzzy integers may be ranked using the space between the centroid and the original location. Deng and Liu [11] recommended using a centroid-index method for sorting fuzzy integers. Additionally, the centroid idea was employed in the ranking indices created by Liang et al. Chinoy & Chinoy. An algorithm was presented in [14] for sorting generalized trapezoidal fuzzy integers. Abbasbandy and Hajjari came up with a new approach based on the left and right spreads at different -levels to rank trapezoidal fuzzy numbers. Fuzzy risk analysis based on ranking generalised fuzzy numbers with varying heights and spreads was presented by Chen and Chen [16].

PRELIMINARIES

In this section some basic definitions, arithmetic operations and ranking function are reviewed.

A.Basic Definitions

In this section some basic definitions are reviewed.

Definition 1. [4] The characteristic function μA of a crisp set $A \subseteq X$ assigns a value either 0 or 1 to each member in X. This function can be generalized to a function $\mu A\tilde{}$ such that the value assigned to the element of the universal set X fall within a specified range i.e. $\mu A\tilde{} : X \rightarrow [0, 1]$. The assigned value indicate the membership grade of the element in the set A.

The function $\mu A\tilde{}$ is called the membership function and the

set $\tilde{A} = \{(x, \mu_{\tilde{A}}(x));$
$x \in X\}$ defined by $\mu_{\tilde{A}}(x)$ for each $x \in X$ is called a fuzzy set.

Definition 2. [4] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be a fuzzy number if its membership function has the following characteristics:
1. $\mu_{\tilde{A}} : R \longrightarrow [0,1]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = 1$ for all $x \in [b,c]$, where $a < b < c < d$.

Definition 3. [4] A fuzzy number $\tilde{A} = (a,b,c,d)$ is said to be a trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} \frac{(x-a)}{(b-a)}, & a < x < b \\ 1 & b \le x \le c \\ \frac{(x-d)}{(c-d)} & c < x < d \end{cases}$$

Definition 4. [16] A fuzzy set $\tilde{A}$, defined on the universal set of real numbers R, is said to be generalized fuzzy number if its membership function has the following characteristics:
1. $\mu_{\tilde{A}} : R \longrightarrow [0,w]$ is continuous.
2. $\mu_{\tilde{A}}(x) = 0$ for all $x \in (-\infty, a] \bigcup [d, \infty)$.
3. $\mu_{\tilde{A}}(x)$ strictly increasing on $[a,b]$ and strictly decreasing on $[c,d]$.
4. $\mu_{\tilde{A}}(x) = w$, for all $x \in [b,c]$, where $0 < w \le 1$.

Definition 5. [17] A fuzzy number $\tilde{A} = (a,b,c,d;w)_{LR}$ is said to be a L-R type generalized fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} wL(\frac{b-x}{b-a}), & \text{for } a < x < b \\ w & \text{for } b \le x \le c \\ wR(\frac{x-c}{d-c}) & \text{for } c < x < d. \end{cases}$$

where L and R are reference functions.

Definition 6. [17] A L-R type generalized fuzzy number $\tilde{A} = (a,b,c,d;w)_{LR}$ is said to be a generalized trapezoidal fuzzy number if its membership function is given by

$$\mu_{\tilde{A}}(x) = \begin{cases} w\frac{(x-a)}{(b-a)}, & a < x < b \\ w & b \le x \le c \\ w\frac{(x-d)}{(c-d)} & c < x < d \end{cases}$$

B. Arithmetic operations

In this subsection, arithmetic operations between two generalized trapezoidal fuzzy numbers, defined on universal set of real numbers R, are reviewed [16].

Let $\tilde{A}_1 = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{A}_2 = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A}_1 \oplus \tilde{A}_2 = (a_1+a_2, b_1+b_2, c_1+c_2, d_1+d_2; min\,(w_1, w_2))$
(ii) $\tilde{A}_1 \ominus \tilde{A}_2 = (a_1-d_2, b_1-c_2, c_1-b_2, d_1-a_2; min\,(w_1, w_2))$
(iii) $\lambda \tilde{A}_1 = \begin{cases} (\lambda a_1, \lambda b_1, \lambda c_1, \lambda d_1; w_1) & \lambda > 0 \\ (\lambda d_1, \lambda c_1, \lambda b_1, \lambda a_1; w_1) & \lambda < 0. \end{cases}$

C. Ranking function

An efficient approach for comparing the fuzzy numbers is by the use of a ranking function [2], _ : $F(R) \rightarrow R$,

where $F(R)$ is a set of fuzzy numbers defined on set of real numbers, which maps each fuzzy number into the real line, where a natural order exists i.e.,

(i) $\tilde{A} \succ \tilde{B}$ iff $\Re(\tilde{A}) > \Re(\tilde{B})$
(ii) $\tilde{A} \prec \tilde{B}$ iff $\Re(\tilde{A}) < \Re(\tilde{B})$
(iii) $\tilde{A} \sim \tilde{B}$ iff $\Re(\tilde{A}) = \Re(\tilde{B})$

Remark 1. [18] For all fuzzy numbers $\tilde{A}, \tilde{B}, \tilde{C}$ and $\tilde{D}$ we have

(i) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{C}$
(ii) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{C} \succ \tilde{B} \ominus \tilde{C}$
(iii) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \oplus \tilde{C} \sim \tilde{B} \oplus \tilde{C}$
(iv) $\tilde{A} \succ \tilde{B}, \tilde{C} \succ \tilde{D} \Rightarrow \tilde{A} \oplus \tilde{C} \succ \tilde{B} \oplus \tilde{D}$

III. SHORTCOMINGS OF CHEN AND CHEN APPROACH

In this section, the shortcomings of Chen and Chen approach [16], on the basis of reasonable properties of fuzzy quantities [18] and on the basis of height of fuzzy numbers, are pointed out

On the basis of reasonable properties of fuzzy quantities Let ˜ A and ˜B be any two fuzzy numbers then

$\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$ (Using remark 1)
i.e., $\Re(\tilde{A}) > \Re(\tilde{B}) \Rightarrow \Re(\tilde{A} \ominus \tilde{B}) > \Re(\tilde{B} \ominus \tilde{B})$

In this subsection, several examples are choosen to prove that the ranking function proposed by Chen and Chen does not satisfy the reasonable property, $\tilde{A} \succ \tilde{B} \Rightarrow (\tilde{A} \ominus \tilde{B}) \succ (\tilde{B} \ominus \tilde{B})$, for the ordering of fuzzy quantities i.e., according to Chen Chen approach $\tilde{A} \succ \tilde{B} \nRightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$, which is contradiction according to Wang and Kerre [18].

Example 1. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

Example 2. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

Example 3. Let $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $\tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{A} \succ \tilde{B}$ but $\tilde{A} \ominus \tilde{B} \prec \tilde{B} \ominus \tilde{B}$ i.e., $\tilde{A} \succ \tilde{B} \nRightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$.

Example 4. Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{B} \succ \tilde{A}$ but $\tilde{B} \ominus \tilde{A} \prec \tilde{A} \ominus \tilde{A}$ i.e., $\tilde{B} \succ \tilde{A} \nRightarrow \tilde{B} \ominus \tilde{A} \succ \tilde{A} \ominus \tilde{A}$.

On the basis of height of fuzzy numbers

Chen and Chen method [16] asserts that the ordering of fuzzy numbers relies on the height of fuzzy numbers in certain circumstances, although this is not always the case, as shown in this part.

Let $\tilde{A} = (a_1, a_2, a_3, a_4; w_1)$ and $\tilde{B} = (a_1, a_2, a_3, a_4; w_2)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen [16] there may be two cases

Case (i) If $(a_1 + a_2 + a_3 + a_4) \neq 0$ then

$$\begin{cases} \tilde{A} \prec \tilde{B}, & \text{if } w_1 < w_2 \\ \tilde{A} \succ \tilde{B}, & \text{if } w_1 > w_2 \\ \tilde{A} \sim \tilde{B}, & \text{if } w_1 = w_2. \end{cases}$$

Case (ii) If $(a_1 + a_2 + a_3 + a_4) = 0$ then $\tilde{A} \sim \tilde{B}$ for all values of w_1 and w_2.

Fuzzy numbers are ranked according to height in the first instance and not according to height at all in the second case, which is a contradiction, according to Chen and Chen [16].

Example 5. Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers then according to Chen and Chen approach $\tilde{A} \prec \tilde{B}$ if $w_1 < w_2$, $\tilde{A} \succ \tilde{B}$ if $w_1 > w_2$ and $\tilde{A} \sim \tilde{B}$ if $w_1 = w_2$.

Example 6. Let $\tilde{A} = (-.4, -.2, -.1, .7; w_1)$ and $\tilde{B} = (-.4, -.2, -.1, .7; w_2)$, be two generalized trapezoidal fuzzy numbers then $\tilde{A} \sim \tilde{B}$ for all values of w_1 and w_2.

IV. PROPOSED APPROACH

In this section, a new approach is proposed for the ranking of generalized trapezoidal fuzzy numbers

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then

(i) $\tilde{A} \succ \tilde{B}$ if RM($\tilde{A}$) >RM($\tilde{B}$)
(ii) $\tilde{A} \prec \tilde{B}$ if RM($\tilde{A}$) <RM($\tilde{B}$)
(iii) $\tilde{A} \sim \tilde{B}$ if RM($\tilde{A}$) =RM($\tilde{B}$) (1)

A. *Method to find values of RM($\tilde{A}$) and RM($\tilde{B}$)*

Let $\tilde{A} = (a_1, b_1, c_1, d_1; w_1)$ and $\tilde{B} = (a_2, b_2, c_2, d_2; w_2)$ be two generalized trapezoidal fuzzy numbers then use the following steps to find the values of RM($\tilde{A}$) and RM($\tilde{B}$)

Step 1 Find $w = min(w_1, w_2)$

Step 2 Find $\Re(\tilde{A}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\}dx$, where $L^{-1}(x) = a_1 + \frac{(b_1-a_1)}{w}x$, $R^{-1}(x) = c_1 + \frac{(b_1-c_1)}{w}x$ $\Rightarrow \Re(\tilde{A}) = \frac{w(a_1+b_1+c_1+d_1)}{4}$ and

$\Re(\tilde{B}) = \frac{1}{2}\int_0^w \{L^{-1}(x) + R^{-1}(x)\}dx$, where $L^{-1}(x) = a_2 + \frac{(b_2-a_2)}{w}x$,
$R^{-1}(x) = c_2 + \frac{(b_2-c_2)}{w}x \Rightarrow \Re(\tilde{B}) = \frac{w(a_2+b_2+c_2+d_2)}{4}$.

Step 3 If $\Re(\tilde{A}) \neq \Re(\tilde{B})$ then RM$(\tilde{A}) = \Re(\tilde{A})$ and RM $(\tilde{B}) = \Re(\tilde{B})$
otherwise RM$(\tilde{A})$ =mode$(\tilde{A}) = \frac{1}{2}\int_0^w b_1 dx + \frac{1}{2}\int_0^w c_1 dx = \frac{w(b_1+c_1)}{2}$ and RM$(\tilde{B})$ =mode$(\tilde{B}) = \frac{1}{2}\int_0^w b_2 dx + \frac{1}{2}\int_0^w c_2 dx = \frac{w(b_2+c_2)}{2}$

Remark 2

Two fuzzy numbers may be joined by using the -cut technique [4] to get arithmetic operations between them, and the highest value of, which is common to both fuzzy numbers, can be found by determining w1 and w2's minimum heights.

V. RESULTS AND DISCUSSION

Fuzzy numbers described in section 3 are correctly arranged in this part. A ranking function suggested here meets the fuzzy quantity features provided by Wang and Kerre [18] as shown in Table 1.

Example 7. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 1)$ and $\tilde{B} = (0.2, 0.3, 0.3, 0.4; 1)$ be two generalized trapezoidal fuzzy numbers
Step 1 $min(1, 1) = 1$
Step 2 $\Re(\tilde{A}) = 0.3$ and $\Re(\tilde{B}) = 0.3$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0.3$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0.3$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 8. Let $\tilde{A} = (0.1, 0.3, 0.3, 0.5; 0.8)$ and $\tilde{B} = (0.1, 0.3, 0.3, 0.5; 1)$ be two generalized trapezoidal fuzzy numbers
Step 1 $min(0.8, 1) = 0.8$
Step 2 $\Re(\tilde{A}) = 0.24$ and $\Re(\tilde{B}) = 0.24$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0.24$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0.24$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 9. Let $\tilde{A} = (-0.8, -0.6, -0.4, -0.2; 0.35)$ and $\tilde{B} = (-0.4, -0.3, -0.2, -0.1; 0.7)$ be two generalized fuzzy numbers
Step 1 $min(0.35, 0.7) = 0.35$
Step 2 $\Re(\tilde{A}) = -0.175$ and $\Re(\tilde{B}) = -0.0875$. Since $\Re(\tilde{A}) \neq \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A}) = \Re(\tilde{A})$ and RM$(\tilde{B}) = \Re(\tilde{B})$. Now RM$(\tilde{A}) <$ RM$(\tilde{B}) \Rightarrow \tilde{A} \prec \tilde{B}$.

Example 10. Let $\tilde{A} = (0.2, 0.4, 0.6, 0.8; 0.35)$ and $\tilde{B} = (0.1, 0.2, 0.3, 0.4; 0.7)$ be two generalized trapezoidal fuzzy numbers then
Step 1 $min(0.35, 0.7) = 0.35$
Step 2 $\Re(\tilde{A}) = 0.175$ and $\Re(\tilde{B}) = 0.0875$. Since $\Re(\tilde{A}) \neq \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A}) = \Re(\tilde{A})$ and RM$(\tilde{B}) = \Re(\tilde{B})$. Now RM$(\tilde{A}) >$ RM$(\tilde{B}) \Rightarrow \tilde{A} \succ \tilde{B}$.

Example 11. Let $\tilde{A} = (1, 1, 1, 1; w_1)$ and $\tilde{B} = (1, 1, 1, 1; w_2)$ be two generalized trapezoidal fuzzy numbers.
Step 1 $min(w_1, w_2) = w$ (say)
Step 2 $\Re(\tilde{A}) = w$ and $\Re(\tilde{B}) = w$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = w$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = w$. Now RM$(\tilde{A}$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

Example 12. Let $\tilde{A} = (-.4, -.2, -.1, .7; w_1)$ and $\tilde{B} = (-.4, -.2, -.1, .7; w_2)$, be two generalized fuzzy numbers then
Step 1 $min(w_1, w_2) = w$ (say)
Step 2 $\Re(\tilde{A}) = 0$ and $\Re(\tilde{B}) = 0$. Since $\Re(\tilde{A}) = \Re(\tilde{B}) \Rightarrow$ RM$(\tilde{A})$ = mode$(\tilde{A}) = 0$ and RM$(\tilde{B})$ = mode$(\tilde{B}) = 0$. Now RM$(\tilde{A})$ = RM$(\tilde{B}) \Rightarrow \tilde{A} \sim \tilde{B}$.

A. Validation of the results

In the above examples it can be easily check that

(i) $\tilde{A} \sim \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \sim \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$ = RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

(ii) $\tilde{A} \succ \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \succ \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B})) \succ$ RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

(iii) $\tilde{A} \prec \tilde{B} \Rightarrow \tilde{A} \ominus \tilde{B} \prec \tilde{B} \ominus \tilde{B}$.
i.e., RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B})) \prec$ RM$((\tilde{A} \ominus \tilde{B}) \ominus (\tilde{B} \ominus \tilde{B}))$

B. Validation of the proposed ranking function

For the validation of the proposed ranking function, in Table 1, it is shown that proposed ranking function satisfies the all reasonable properties of fuzzy quantities proposed by Wang and Kerre [18].

TABLE I
FULFILMENT OF THE AXIOMS FOR THE ORDERING IN THE FIRST AND SECOND CLASS [18]

Index	A_1	A_2	A_3	A_4	A'_4	A_5	A_6	A'_6	A_7
Y_1	Y	Y	Y	Y	Y	Y	N	N	N
Y_2	Y	Y	Y	Y	Y	Y	Y	Y	N
Y_3	Y	Y	Y	N	N	Y	N	N	N
Y_4	Y	Y	Y	Y	Y	Y	N	N	N
C	Y	Y	Y	N	N	Y	N	N	N
FR	Y	Y	Y	Y	Y	Y	Y	Y	N
CL	Y	Y	Y	Y	Y	Y	Y	Y	N
LW^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_1^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
CM_2^{λ}	Y	Y	Y	Y	Y	Y	Y	Y	N
K	Y	Y	Y	N	N	N	N	N	N
W	Y	Y	Y	Y	N	N	N	N	N
J^k	Y	Y	Y	Y	Y	N	N	N	N
CH^k	Y	Y	Y	Y	Y	N	N	N	N
KP^k	Y	Y	Y	Y	Y	N	N	N	N
Proposed Approach	Y	Y	Y	Y	Y	Y	Y	Y	N

VI. CONCLUSION

A novel ranking method for obtaining the right order of generalised trapezoidal fuzzy numbers is provided in this study, highlighting the inadequacies of Chen and Chen [16]. The suggested ranking function meets all of the acceptable features of fuzzy quantities established by Wang and Kerre [18] as shown.

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